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AUTHOR'S CLOSURE

Firstly, it is necessary to clear up some misconceptions which may have arisen as a result of the discussion. The expressions for the shear stiffness are not developed on the basis of any assumptions about fixed support conditions. However, they are implicitly invoked in the development of the slope-deflection equations in Section 4. The nature of support conditions will be discussed in more detail later. It is not suggested that the shear behaviour is a second approximation to the full behaviour of a beam in some kind of asymptotic expansion. What has been done is to classify six modes of response to the six components of resultant load that can be applied to a prismatic system (which could be a bar or a truss). In this context, it is not possible to say that "in general" the shear deflection is two orders smaller than the bending deflection, although this can be true in specific instances. For example, using the expressions similarly derived for the shear and bending stiffnesses of a vierendeel truss, it can be shown (Renton, 1984) that the shear deformation (racking) of a multi-storey portal frame is typically more significant than its bending deformation for up to forty-storey systems, as suggested by Lin and Stotesbury (1981).

The problem examined in the discussion is not a true three-dimensional representation of a rectangular beam, but a plane stress approximation to it. Likewise, the shear stiffnesses listed in the introduction result from attempts to reduce the problem to two-dimensional analyses. In this sense, they are similar in that they take no account of the breadth to depth ratio of the beam and do not differ from one another by more than 50%. The present analysis does take account of this ratio and predicts that the shear stiffness of broad plank-like sections is very much less than was previously thought to be the case.

According to Saint-Venant's principle, the only non-decaying stress and strain responses to end loading are the six listed above. The deflected state is not determined entirely by these responses, because rigid-body motions can also occur. These will in turn depend on the way in which the system is supported or connected. If the support conditions are related to an end displacement or rotation, these deflections must be clearly defined. It might appear reasonable to take an average displacement or rotation of the end cross-section or to use the deflections of the centroid, as for example in Timoshenko and Goodier (1970) Fig. 27b. However, it is questionable whether in practice a fixed support would permit displacements of individual points on the end cross-section while preventing the average displacement or rotation, or that at some specific point. This becomes even more questionable when it is applied to inhomogeneous beams or to trusses.

The paper uses work and energy principles which are fundamental to the theory of elasticity. These can also be used in considering support conditions. The end reactions can do no work, otherwise they cease to be reactions. If this were not so, the thermodynamic principle that there are no free lunches is flouted and perpetual-motion machines become possible. Work can be done on the end supports, but then the elastic or load-deflection properties of the supports must be specified. This leaves the class of workless reactions most commonly used in engineering analyses. These are such that either any component of the resultant end load is zero or the corresponding deflection through which it does work is zero. This deflection may or may not be some average deflection or that of the centroid of the section. However, it can be determined in terms of energy principles, as in Section 4 of the paper. A fixed end is then one in which all these corresponding deflections are zero. Pinned ends and roller bearings also correspond to the class of workless reactions, as does a free end. Indeed, the shear condition at the free edge of a plate is best understood in terms of the expressions for work and energy.

In using the engineering theory of beams, it must be borne in mind that only some form of macroscopic behaviour is being analysed. The governing differential equation for flexure is fourth order, leaving only two conditions to be specified at each end. These will usually be a moment/rotation condition and a shear force/displacement condition. Likewise, the usual equation for torsion is second order, permitting only one torque/twist condition to be imposed at each end. However, if warping is constrained at the ends of a thin-walled bar of open cross-section, the apparent torsional stiffness of the bar can be much greater than that predicted by the usual Saint-Venant theory of torsion, as was mentioned in the discussion. This effect has been extensively examined by Vlasov, Gorbunov and Strelbitskaya, Stavraki and others [see for example Renton (1974)]. The method adopted by these authors is not to discard the sectional torsional stiffness as meaningless, or replace it with a value which depends on the end conditions as well as the sectional properties. Instead, they examine the response of the section to a warping stress system, known as a bimoment, which has no resultant moment or force. This involves the introduction of a further sectional property known as the sectorial moment of inertia or non-uniform torsional constant. The second-order equation now becomes a fourth-order equation which now permits a second (bimoment/warping) condition to be imposed at each end. Likewise, if more accurate analyses of the flexural/shear behaviour of beams are required, it would seem appropriate to approach the problem by examining the response of a beam to end stress systems which have no resultants but distort the cross-section. Then the existing differential equation would similarly be augmented rather than replaced.

Finally, the problem of a concentrated couple represents no paradox, but in fact illustrates the need to consider the shear behaviour of a beam as well as its response to a moment. As the two forces approach one another, growing inversely with their separation, so does the shear strain induced between them, until eventually the material will fail. This effect can readily be demonstrated experimentally with a pair of scissors.

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REFERENCES

Lin, T. Y. and Stotesbury, S. D. (1981). *Structural Concepts and Systems for Architects and Engineers*. Wiley, New York.

Renton, J. D. (1974). On the transmission of non-uniform torsion through joints. O.U. Engineering Laboratory Report No. 1086/74. Oxford, U.K.

Renton, J. D. (1984). Characteristic properties of regular structures. *Proceedings, International Conference on Steel Structures*, March 1984. Singapore.

Timoshenko, S. P. and Goodier, J. N. (1970). *Theory of Elasticity*. McGraw-Hill, Maidenhead, U.K.